Question 3

A cubic function \( f \) is defined for \( x \in \mathbb{R} \) as
\[
f : x \mapsto x^3 + (1-k^2)x + k,
\]
where \( k \) is a constant.

(a) Show that \( -k \) is a root of \( f \).

(b) Find, in terms of \( k \), the other two roots of \( f \).

(c) Find the set of values of \( k \) for which \( f \) has exactly one real root.
Question 2

(a) Solve the simultaneous equations:

\[ 2x + 8y - 3z = -1 \]
\[ 2x - 3y + 2z = 2 \]
\[ 2x + y + z = 5. \]

(b) The graphs of the functions \( f : x \mapsto |x - 3| \) and \( g : x \mapsto 2 \) are shown in the diagram.

(i) Find the co-ordinates of the points \( A, B, C \) and \( D \).

\[ A = (\text{ }, \text{ }) \]
\[ B = (\text{ }, \text{ }) \]
\[ C = (\text{ }, \text{ }) \]
\[ D = (\text{ }, \text{ }) \]

(ii) Hence, or otherwise, solve the inequality \( |x - 3| < 2 \).
Question 3

(a) Find the set of all real values of $x$ for which $2x^2 + x - 15 \geq 0$.

(b) Solve the simultaneous equations:

\[
\begin{align*}
    x + y + z &= 16 \\
    \frac{5}{2} x + y + 10z &= 40 \\
    2x + \frac{1}{2} y + 4z &= 21.
\end{align*}
\]
A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1 reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

(a) If the ticket price was €18, how many people would be expected to attend?

(b) Let $x$ be the ticket price, where $x \leq 20$. Write down, in terms of $x$, the expected attendance at such an event.

(c) Write down a function $f$ that gives the expected income from the sale of tickets for such an event.

(d) Find the price at which tickets should be sold to give the maximum expected income.
(e) Find this maximum expected income.

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at (e) above.

(g) The stadium was full for a recent special event. Two types of tickets were sold, a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold, the income from the event would have been reduced by €14 000. How many family tickets were sold?
Question 5
Question 4

(a) Solve the simultaneous equations,

\[ 2x + 8y - 3z = -1 \]
\[ 2x - 3y + 2z = 2 \]
\[ 2x + y + z = 5. \]
Question 1

(a) Solve the simultaneous equations:

\[ a^2 - ab + b^2 = 3 \]
\[ a + 2b + 1 = 0 \]

(b) Find the set of all real values of \( x \) for which \( \frac{2x - 5}{x - 3} \leq \frac{5}{2} \).